Things you will need to know for the Real Analysis in MATH20101

Factorisation

In the course we will have need occasionally to factorise a polynomial. We will only be interested in polynomials with real coefficients so the problem is to factorise over $\mathbb{R}$, i.e. find factor polynomials with real coefficients.

Recall that if a polynomial $p(x)$ has a root a then $x-a$ is a factor of $p(x)$, i.e. there exists a polynomial $q(x)$ such that $p(x) = (x-a)\,q(x)$. If a is real then the coefficients of $q(x)$ will be real.

1) For $x, y \in \mathbb{R}$ factor

i) $x^2 - y^2$

ii) $x^3 - y^3$,

iii) $x^n - y^n$, where $n \geq 1$.

2) Factorise the following:

i) (2004) $2x^2 - x - 6$,

ii) (2007) $x^2 + x - 2$,

iii) (2008) $x^2 + x - 12$,

iv) (2008) $x^3 - 27$,

v) (2009) $x^3 + x - 30$..

3) For more examples, not seen in any exam papers, try factorising

i) $p(x) = x^4 + x^3 - 2x^2 - 6x - 4$,

Hint, look for small integer roots.

ii) (Harder) $p(x) = 6x^3 + 27x^2 + 37x + 15$

Hint: this polynomial does **not** have an *integer* root as in the last example, and so does **not** have a factor of the form $x + a$ for some $a \in \mathbb{Z}$. But it **does** have a factor of the form $ax + b$ for some $a, b \in \mathbb{Z}$. Show that for such a factor we must have $a|6$ (the coefficient of x^3) and $b|15$ (the constant term).

Search through all possible such (a, b) to find a factorization of $p(x)$ into a linear and a quadratic factor, both factors having integer coefficients.

iii) $p(x) = 6x^4 + 39x^3 + 91x^2 + 89x + 30$.

Hint this does have an integer root.

ii) (Harder) $p(x) = x^4 + 4x^3 + 11x^2 + 14x + 12$.

Hint, this polynomial does factor but has no real *linear* factor but does factor into two quadratics, each with integer coefficients.

For more information on polynomials see WEB ADDRESS